

# 41<sup>st</sup> Annual Dick Schaff Math Superbowl

## Level 6 (Calculus) Huddle

Directions: Select the most correct answer for each question and mark it on your Scantron® sheet. N.O.T. stands for "None of These."

1. Let  $P(x) = x^7 + a_6x^6 + a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$ , where all  $a_i$  are Real numbers. What is the maximum number of turning points  $P(x)$  can have?

- A) 2                      B) 3                      C) 4                      D) 5                      E) N.O.T.

2. Evaluate  $\lim_{x \rightarrow 0} (5)$ .

- A) 0                      B) 5                      C) Infinity                      D) Does Not Exist                      E) N.O.T.

3. Let  $Q(x) = x^5 + 3x + 1$ . According to the Intermediate Value Theorem, on which of the following intervals could  $Q(x)$  have a root?

- A)  $[-1, 0]$                       B)  $[0, 1]$                       C)  $[1, 2]$                       D)  $[2, 3]$                       E) N.O.T.

4. Let  $f(x) = xe^x$ . Find  $f^{(2014)}(x)$ .

- A)  $xe^x$                       B)  $x^{2014}e^{2014x}$                       C)  $2013e^x + xe^x$                       D)  $2014e^x + xe^x$                       E) N.O.T.

5. Evaluate  $\lim_{x \rightarrow \infty} (\sqrt{x^2 - 7x} - \sqrt{x^2 + 7x})$ .

- A) 7                      B) -7                      C) Infinity                      D) Does Not Exist                      E) N.O.T.

6. Let  $g(x) = (x + a)(x + b)$ , where  $a$  and  $b$  are real numbers. For what value of  $x$  is  $g'(x) = 0$ ?

- A)  $a$                       B)  $b$                       C)  $a + b$                       D)  $a - b$                       E) N.O.T.

7. If air resistance is negligible, and acceleration due to gravity is  $-32$  feet/second<sup>2</sup>, what is the initial velocity of a penny that hits the ground five seconds after being tossed off a 500 foot tall observation deck?

- A) 20 ft/sec      B)  $-20$  ft/sec      C) 40 ft/sec      D)  $-40$  ft/sec      E) N.O.T.

8. Let  $y = \cos^2(x)$ . Find  $d^2y/dx^2$ .

- A)  $2\sin^2(x) - 2\cos^2(x)$   
B)  $2\sin^2(x) + 2\cos^2(x)$   
C)  $-2\sin^2(x) + 2\cos^2(x)$   
D)  $-2\sin^2(x) - 2\cos^2(x)$   
E) N.O.T.

9. Let  $f''(x) = 6x + \sin(x)$ . Which of the following could be  $f(x)$ ?

- A)  $x^3 + \sin(x)$       B)  $x^3 + x + \sin(x)$       C)  $x^3 + 1 + \sin(x)$       D)  $x^3 + x + 1 + \sin(x)$       E) N.O.T.

10. What is the area bound between the curves  $y_1 = x^2 - 4x$  and  $y_2 = 2x + 16$ ?

- A) 0      B) 316      C) 320      D) 324      E) N.O.T.

11. Let  $y = 5x + 6$ . Which of the following intervals for  $x$  guarantees  $15.9 < y < 16.1$ ?

- A)  $1.8 < x < 2.2$       B)  $1.9 < x < 2.1$       C)  $1.95 < x < 2.05$       D)  $1.98 < x < 2.02$       E) N.O.T.

12. Evaluate  $\lim_{x \rightarrow 0} \left( \frac{\sin(2x)}{\sin(5x)} \right)$ .

- A) 2      B) 5      C) 0.4      D) 2.5      E) N.O.T.

13. What is the area under the curve  $y = 10^x$  between  $x = 1$  and  $x = 2$ ?

- A)  $\frac{90}{\ln(10)}$       B)  $\frac{99}{\ln(10)}$       C)  $90 \ln(10)$       D)  $99 \ln(10)$       E) N.O.T.

14. Let  $x^2 + 4xy + 2y^2 = 1$ . Find  $dy/dx$ .

- A)  $\frac{x+2y}{2x+2y}$       B)  $\frac{x-2y}{2x+2y}$       C)  $\frac{-x+2y}{2x+2y}$       D)  $\frac{-x-2y}{2x+2y}$       E) N.O.T.

15. Let  $xy = k$ , where  $k$  is a Real number. Find  $d^2y/dx^2$ .

- A)  $2kx^3$       B)  $2kx^{-3}$       C)  $-2kx^3$       D)  $-2kx^{-3}$       E) N.O.T.

16. Evaluate  $\lim_{x \rightarrow \infty} (5)$ .

- A) 0      B) 5      C) Infinity      D) Does Not Exist      E) N.O.T.

17. For which of the following functions is  $f(-x) = -f(x)$ ?

- A)  $f(x) = \cos(x)$       B)  $f(x) = e^x$       C)  $f(x) = \ln(x)$       D)  $f(x) = \tan(x)$       E) N.O.T.

18. A sphere with radius 2 m is coated with paint 0.5 mm thick. What is a reasonable approximation for the volume of the paint used?

- A)  $8\pi \text{ mm}^3$       B)  $8,000\pi \text{ mm}^3$       C)  $8,000,000\pi \text{ mm}^3$       D)  $0.008\pi \text{ mm}^3$       E) N.O.T.

19. Let  $g(x) = \frac{x-8}{x^2}$ . Where does  $g(x)$  have a local maximum?

- A) 0      B) 8      C) 16      D) 24      E) N.O.T.

20. Let  $P(x) = \int_0^{6x} e^{2t} dt$ . Find  $dP/dx$ .

- A)  $e^{6x}$                       B)  $2e^{6x}$                       C)  $3e^{6x}$                       D)  $6e^{6x}$                       E) N.O.T.

21. Evaluate  $\lim_{x \rightarrow 0} \left( \frac{\sin(3x) - 3x}{\sin(5x) - 5x} \right)$ .

- A) 0                      B) 0.6                      C) 0.36                      D) 0.216                      E) N.O.T.

22. Let  $f(x) = (P \circ Q)(x)$ . Find  $f'(x)$ .

- A)  $(P' \circ Q')(x)$               B)  $(P' \circ Q)(x)$               C)  $(P' \circ Q')(x)Q'(x)$       D)  $(P' \circ Q)(x)Q'(x)$       E) N.O.T.

23. Let  $y = \sin^2(3x)$ . Find  $dy/dx$ .

- A)  $2\sin(3x)$               B)  $6\sin(3x)$               C)  $2\cos(3x)$               D)  $6\cos(3x)$               E) N.O.T.

24. What is the slope of the line tangent to  $xy = 8$  at the point  $(2, 4)$ ?

- A) 2                      B) -2                      C) 4                      D) -4                      E) N.O.T.

25. The area of a circular oil spill is growing at a rate of  $20 \text{ m}^2/\text{second}$ . How fast is the radius of this oil spill growing when the radius is  $100 \text{ m}$ ?

- A)  $\frac{1}{10\pi} \text{ m/second}$       B)  $\frac{1}{20\pi} \text{ m/second}$       C)  $\frac{1}{50\pi} \text{ m/second}$       D)  $\frac{1}{100\pi} \text{ m/second}$       E) N.O.T.